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9. Canonical Correlation and Marketing Research

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INTRODUCTION

In many situations, marketing researchers and decision makers have an interest in determining the relationship between indices of variables rather than the original variables themselves. This occurs, for example, where relations between behavioral constructs such as attitudes and purchase intentions towards brands are under scrutiny, and there are several measures on each construct from which indices can be produced. Similarly, the marketing manager's objective function may be some index created from variables, such as sales, market share, image, etc., which are determined by indices of marketing mix, such as advertising, price, and other marketing mix variables. A third example is provided by the several effects of an advertisement upon awareness, liking, etc., and the relationship of these effects to the dimensions defining the advertisement, such as size, color, placement, media, etc. (Sheth, 1971a). In such situations, the canonical correlation technique might be used to provide insights into the nature and magnitude of the relationships between sets of variables.

Although most often confined to the analysis of the relationship between two sets of variables, canonical correlation extends more or less directly to several sets and their interrelationships. This would be of interest, for example, when the perceptions of different groups of people (customers, suppliers, employees, stockholders) of a company's market performance are compared, and performance is measured by a set of separate variables. Similarly, different segments might be identified and their characteristics compared and related to company and product characteristics. Another example would be the characterization of competitors and the researcher's own company along certain marketing variables attempting to isolate some typical patterns of actions and reactions, and the consequent payoffs. Also, the case dealt with above in which multiple objectives are related to controllable marketing variables could be augmented with sets of uncontrollables, such as market conditions and competitive actions. In short, the potential of two set and multiple-set canonical theory, as applied to these types of marketing problems, seems very great.

TWO-SET CANONICAL CORRELATION

Consider two *sets* of variates with a joint distribution. Our objective is to analyze the correlations between the variables of one set and those of the other set. We find a new coordinate system in the space of each set of variables in such a way that the new coordinates display the system of correlations unambiguously. This means we must find the linear combinations or indices of variables in each set that have the *maximum* correlation. They constitute the first coordinates in the new system. Then a second linear combination in each set is sought whose correlation is the maximum of all correlations between any two linear combinations which are uncorrelated with the first linear combinations. The procedure is continued until the new coordinate system is completely specified (Anderson, 1958).

Procedure

Consider two sets of N simultaneous equations with p predictors and q criterion variables where x_{ij} and y_{ij} represent the two sets of measures:

$$\hat{X}_1 = a_1x_{11} + a_2x_{12} + \dots + a_px_{1p}; \quad b_1y_{11} + b_2y_{12} + \dots + b_qy_{1q} = \hat{Y}_1$$

$$\hat{X}_2 = a_1x_{21} + a_2x_{22} + \dots + a_px_{2p}; \quad b_1y_{21} + b_2y_{22} + \dots + b_qy_{2q} = \hat{Y}_2$$

$$\vdots$$

$$\hat{X}_N = a_1x_{N1} + a_2x_{N2} + \dots + a_pX_{Np}; \quad b_1y_{N1} + b_2y_{N2} + \dots + b_qy_{Nq} = \hat{y}_N$$

Find the vectors of weights $\underline{a}$ and $\underline{b}$ that maximize the correlations between $\hat{X}_i$ and $\hat{Y}_i$, distributions of the indices or linear combinations of the observed vectors of responses of an individual observation i in the same sample.

If p = q = 1, then this is simply a problem of simple regression. If q = 1 and p > 1, it is a multiple regression problem. Finally, if p > 1 *and* q > 1 (and assuming for convenience $q \le p$), it is a problem in canonical correlation analysis.

In canonical analysis, the number of possible pairs of linear combinations is p or q, whichever is smaller. Under the assumption that $q \le p$, the number of pairs is generally, therefore, equal to the number of criterion variables. Each pair of canonical variates X_j and Y_j (j = 1...q) is maximally correlated, subject to the restriction that each canonical variate or the linear combination as a coordinate system be orthogonal to all other canonical variables on its side of the equation.

The analysis begins with the partitioning of R, the matrix of intercorrelations for p + q variables:

$$R = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix}$$

where

R_{11} = intercorrelations among p predictor variables

R_{22} = intercorrelations among q criterion variables

R_{12} = R_{21} = intercorrelations between p predictor and q criterion variables.

From this information the following canonical equations are derived:

$$(R_{22}^{-1} R_{21} R_{11}^{-1} R_{12} - \lambda_j I) \underline{b}_j = 0$$

and

$$(R_{11}^{-1} R_{12} R_{22}^{-1} R_{21} - \lambda_j I) \underline{a}_j = 0$$

These equations conform to the invariant linear transformation specified by the theorem of characteristic equations. Therefore, the solution involves defining characteristic roots λ for which the determinants are zero:

$$|R_{22}^{-1} R_{21} R_{11}^{-1} R_{12} - \lambda I| = 0$$

$$|R_{11}^{-1} R_{12}^{-1} R_{22} R_{21} - \lambda I| = 0$$

If, for some reason, q > p there are q possible roots but q - p of these are equal to zero.

The characteristic vectors associated with each λ_j then become the vector of coefficients $\underline{b}_j$ and $\underline{a}_j$. These are also defined interchangeably as follows:

$$\underline{b}_j = (R_{12} R_{11}^{-1} \underline{a}_j) / \sqrt{\lambda_j}$$

and

$$\underline{a}_j = (R_{11}^{-1} R_{12} \underline{b}_j) / \sqrt{\lambda_j}$$

This avoids the necessity of solving for two characteristics equations.

The canonical correlation R between the j^{th} pair of new composites is $\sqrt{\lambda_j}$. The largest λ_j is the square of the maximum possible correlation between linear combinations of the two sets of variables. Thus

$$\text{Max } R^2 = \lambda_1$$

Rationale

Let us define the random vector T of t components (t = p + q). It has the covariance matrix Σ which is presumed to be positive semidefinitive. We are only interested in variances and covariances and therefore we can presume that $\bar{T} = 0$. In developing the concepts and algebra we do not need to assume that T is normally distributed, although presumption of normality will become necessary when developing tests of significance of parameters based on multivariate normal sampling theory.

Partition T into two subvectors of p and q components:

$$T = \begin{bmatrix} P \\ Q \end{bmatrix} \quad (1)$$

The covariance matrix is partitioned similarly into p and q rows and columns.

$$\Sigma_T = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix} \quad (2)$$

Make an arbitrary linear combination, $U = \underline{a}'P$, of the components of P and another arbitrary linear function, $V = \underline{b}'Q$, of the components of Q. We first ask for the linear functions that have maximum correlation. Since the correlation of a multiple of U (such as xU, x = 2) and a multiple of V (such as yV, y = 5) is the same as the correlation of U and V, we can make an arbitrary normalization of $\underline{a}$ and $\underline{b}$. In fact, this becomes mandatory rather than just a desirable feature because of the need for identification of the variates.

We therefore require $\underline{a}$ and $\underline{b}$ to be such that U and V have unit variance:

$$1 = UU'/N = \underline{a}'(P\,P'/N)\underline{a} = \underline{a}'\Sigma_{11}\underline{a} \qquad (3)$$

$$1 = VV'/N = \underline{b}'(Q\,Q'/N)\underline{b} = \underline{b}'\Sigma_{22}\underline{b} \qquad (4)$$

we note that $\bar{U} = \underline{a}'\,\bar{p} = 0$ and similarly $\bar{V} = \underline{b}'\,\bar{Q} = 0$.

The correlation between U and V is simply

$$UV/N = \underline{a}'(PQ/N)\underline{b} = \underline{a}'\Sigma_{12}\underline{b} \qquad (5)$$

The algebraic problem is simply to maximize (5) subject to the normalization conditions specified in (3) and (4).

We can easily construct this constrained maximization problem by defining the function (6) as

$$\Psi = \underline{a}'\Sigma_{12}\underline{b} - [1/2\ \lambda(\underline{a}'\Sigma_{11}\underline{a} - 1)] - [1/2\ u\ (\underline{b}'\Sigma_{22}\underline{b} - 1)] \qquad (6)$$

where λ and u are the Lagrange multipliers. Differentiate (6) with respect to the elements of $\underline{a}$ and $\underline{b}$. The vectors of derivatives set equal to zero are:

$$\delta\Psi/\delta_{\underline{a}} = \Sigma_{12}\underline{b} - \lambda\Sigma_{11}\underline{a} = 0 \qquad (7)$$

$$\delta\Psi/\delta_{\underline{b}} = \underline{a}'\Sigma_{12} - u\underline{b}'\Sigma_{22} = 0 \qquad (8)$$

Multiplying (7) on the left by $\underline{a}'$ and (8) on the right by $\underline{b}$ results in

$$\underline{a}'\Sigma_{12}\underline{b} - \lambda\underline{a}'\Sigma_{11}\underline{a} = 0 \qquad (9)$$

$$\underline{a}'\Sigma_{12}\underline{b} - u\underline{b}'\Sigma_{22}\underline{b} = 0 \qquad (10)$$

Since $\underline{a}'\Sigma_{11}\underline{a} = 1$ and $\underline{b}'\Sigma_{22}\underline{b} = 1$, this shows that $\lambda = u = \underline{a}'\Sigma_{12}\underline{b}$. In other words, the maximum correlation between the two sets of variables P and Q via their linear combinations $U = \underline{a}'P$ and $V = \underline{b}'Q$, is $\underline{a}'\Sigma_{12}\underline{b}$.

Equations (7) and (8) can be rewritten to present a single matrix equation, remembering that $\Sigma_{12} = \Sigma_{21}$ and $\lambda = u$ from equations (9) and (10) above. Thus we get

$$-\lambda\Sigma_{11}\underline{a} + \Sigma_{12}\underline{b} = 0 \qquad (11)$$

$$\Sigma_{21}\underline{a} - \lambda\Sigma_{22}\underline{b} = 0 \qquad (12)$$

The single matrix equation then is

$$\begin{bmatrix} -\lambda\Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & -\lambda\Sigma_{22} \end{bmatrix} \begin{bmatrix} \underline{a} \\ \underline{b} \end{bmatrix} = 0 \qquad (13)$$

For a nontrivial solution, the matrix on the left of (13) must be singular and, therefore, its determinant should be zero:

$$\begin{vmatrix} -\lambda\Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & -\lambda\Sigma_{22} \end{vmatrix} = 0$$

The determinant of this matrix on the left is a polynomial of degree t = p + q, and has t roots, say $\lambda_1 > \lambda_2 \ldots > \lambda_t$. Only the first q roots, however, will be nonzero because of the limit of the rank of the criterion variable matrix which cannot exceed the number q variables.

Since we are interested in the maximum correlation, we take $\lambda = \lambda_1$ because as we saw earlier $\lambda = \underline{a}'\Sigma_{12}\underline{b}$. Let the first solution for equation (13) be $\lambda = \lambda_1$ and the corresponding $\underline{a_1}$ and $\underline{b_1}$ vectors associated with the first root. Now let $U_1 = \underline{a_1}'P$ and $V_1 = \underline{b_1}'Q$. Then U_1 and V_1 are normalized linear combinations of P and Q respectively which have the maximum correlation.

We can solve for either $\underline{a_1}$ or $\underline{b_1}$ from the single matrix equation given in (13). To do this, multiply equation (11) with λ and (12) with Σ_{22}^{-1}. Thus

$\lambda(-\lambda\Sigma_{11}\underline{a} + \Sigma_{12}\underline{b} = 0)$, which can be rewritten as

$$\lambda\Sigma_{12}\underline{b} = \lambda^2\Sigma_{11}\underline{a} \qquad (15)$$

and

$\Sigma_{22}^{-1}(\Sigma_{21}\underline{a} - \lambda\Sigma_{22}\underline{b} = 0)$, which can be rewritten as

$$\Sigma_{22}^{-1}\Sigma_{21}\underline{a} = \lambda\underline{b} \qquad (16)$$

Substitution from (16) into (15) gives

$$\Sigma_{12}\ \Sigma_{22}^{-1}\ \Sigma_{21}\underline{a} = \lambda^2\Sigma_{11}\underline{a}$$

or

$$(\Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21} - \lambda^2\Sigma_{11})\underline{a} = 0 \qquad 17)$$

This can be transformed into the familiar characteristic equation by multiplying it with Σ_{11}^{-1}

$$(\Sigma_{11}^{-1}\Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21} - UI)\underline{a} = 0$$

where $U = \lambda^2$.

The determinant of the equation is

$$|\Sigma_{11}^{-1}\Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21} - UI| = 0$$

and $\underline{a}_1 \ldots \underline{a}_p$ satisfy equation (17) for $\lambda^2 = u_1 \ldots u_p$, respectively.

Similarly, if we premultiply equations (11) and (12) with Σ_{11}^{-1} and λ respectively, we obtain

$$\Sigma_{11}^{-1}\Sigma_{12}\underline{b} = \lambda\underline{a}$$

and

$$\lambda\Sigma_{21}\underline{a} = \lambda^2\Sigma_{22}\underline{b}$$

Substituting the first equation into the second, we get

$$\Sigma_{21}\Sigma_{11}^{-1}\Sigma_{12}\underline{b} = \lambda^2\Sigma_{22}\underline{b}$$

or

$$(\Sigma_{21}\Sigma_{11}^{-1}\Sigma_{12} - \lambda^2\Sigma_{22})\underline{b} = 0 \qquad (18)$$

The characteristic equation for (18) is

$$(\Sigma_{22}^{-1}\Sigma_{21}\Sigma_{11}^{-1}\Sigma_{12} - UI)\underline{b} = 0$$

and $\underline{b}_1 \ldots \underline{b}_q$ vectors corresponding to $u_1 \ldots u_q$ satisfy equation (18).

If we recall the nature of the data, we see that $\bar{T} = 0$ and, therefore, the original observations are in terms of deviation scores rather than raw scores. In addition, we specified in equations (3) and (4), the conditions that UU'/N = 1 and therefore PP'/N = 1. In other words, the data are normalized also. Under those conditions, $\Sigma = R$ and $\Sigma_{11} = R_{11}$, $\Sigma_{22} = R_{22}$ and

$$\Sigma_{12} = \Sigma_{21}' = R_{12} = R_{21}'.$$

We can, therefore rewrite equations (17) and (18) in terms of correlations rather than covariances. In the characteristic equations form, they are:

$$(R_{11}^{-1}R_{12}R_{22}^{-1}R_{21} - UI)\underline{a} = 0 \qquad (19)$$

and

$$(R_{22}^{-1}R_{21}R_{11}^{-1}R_{12} - UI)\underline{b} = 0 \qquad (20)$$

These were precisely the equations described in the earlier section on procedure.

INTERPRETATION OF CANONICAL CORRELATIONS[1]

We will review the interpretation of canonical correlation results in terms of statistical significance, strength of relationships and interrelationship with other techniques, especially with multiple regression analysis.

Several statistics can be used for evaluating how significant the results of the two-set canonical analysis are. As for the canonical roots themselves, the appropriate test is Bartlett's approximate chi-square (Bartlett, 1941), normally printed out in computer applications. The chi-square value is computed as

$$\chi^2 = -[(N-1) - .5(p + q + 1)]\ln\Gamma$$

where

N = # of observations
p = # of predictors
q = # of criterion variables
$(p \geq q)$

and

$$\Gamma = \prod_{i=1}^{q}(1-\lambda_i). \qquad (21)$$

with λ_i^2 the characteristic root associated with the i^{th} variates, i=1, ..., q. The degrees of freedom are equal to p·q.

For the case where one wants to evaluate the significance of each of the roots separately, the appropriate test is based on the same chi-square statistic, but now with

$$\Gamma_i = (1 - \lambda_i),\ i=1, \ldots, q \qquad (22)$$

and degrees of freedom equal to (p - r) + (q - r) - 1, where r is the number of canonical relationships preceding the one tested. Finally, when the interest is in assessing the significance of the roots with, say, the first r removed, the test is identical but now with

$$\Gamma = \prod_{i=r+1}^{q}(1 - \lambda_i) \qquad (23)$$

and degrees of freedom equal to (p - r) + (q - r).

There is as yet no statistical test of the significance of the weights with which the variables enter each variate. This fact has been one of the major stumbling blocks in interpretation, since the researcher will not generally know whether to pay attention to a given weight or not. It is difficult to know when in fact a high weight is really "high" and a low weight "low." An approximate test based upon a multiple correlation correspondence can be developed (Johansson and Sheth, 1973b). For the multiple-set canonical analysis, (see below) no significance tests are currently available.

The Strength of the Canonical Relationships

Turning from the significance to the strength of the relationships uncovered, it should be pointed out that a high significance in itself means very little when it comes to the amount of explication. This is because the canonical correlations refer to the variance explained in the linear composites and not in the original variables. However, measures such as the mean square of the canonical correlation:

$$MSCC = \sum_{i=1}^{q}\lambda_i/q$$

and the total variance extracted:

$$TVE = \sum_{j=1}^{q}\sum_{i=0}^{j-1}\lambda_j(1 - \lambda_i)$$

are to some extent useful for inferring the relative contributions of various canonical variates. Recently attempts have been made to provide insights into the strength of canonical correlation analysis in terms of the total variance of the criterion variables. The statistic now in common use for this purpose is the redundancy index, first proposed by Stewart and Love (1968) and by Miller (1969). We will adopt Alpert and Peterson's (1972) notation in what follows:

Let

VC_i = Proportion of criterion set variance explained by i^{th} variate.
VP_i = Proportion of predictor set variance explained by i^{th} variate.

When we multiply VC_i by λ_i we arrive at a measure of the variance of the criterion variables explained by the correlation between the i'th canonical variates. Summing this value for all the variates generates an index of the proportion of variance in the criteria explicable by the predictors. This measure is called the redundancy index and it can be written:

$$RED_{c/p} = \sum_{i=1}^{q}\lambda_i VC_i = \sum_{i=1}^{q}\lambda_i\left[\sum_{j=1}^{q}\frac{L_{ji}^2}{q}\right]$$

where L_{ji} is the correlation or loading between the j'th criterion variable and the i'th canonical variate.

The total redundancy in the criterion set with respect to the predictor set is equivalent to the mean coefficient of multiple determination between the predictors and each criterion variable.[2]

It seems quite reasonable to use the canonical roots-- for which significance tests are easily available, as we have seen-- to test for the existence of a relationship between the two sets of variates. When it comes to measuring the strength of an existing relationship, however, the redundancy index might be a better measure. Partly because of this advantage of the redundancy index, some work has been done to find the first pair of variates which will maximize the redundancy rather than the canonical correlation (Johansson and Narayan, 1973). The results seem to indicate that the canonical correlation solution in fact often provides a fairly good approximation to the redundancy attributable to the first root. No multiple-set generalization of the redundancy index has yet been proposed.

Even with the basic strength of the relationships between the sets of variables established, however, one would still like to make substantive and statistical interpretations of the results. For this it becomes necessary to go back to the meaning of the original variable measures and their weights in the (significant) canonical variates. Without significance tests of the weights available, however, this part of the interpretation becomes a great stumbling block, as we have mentioned earlier. Some assistance is provided by Alpert and Peterson's (1972) proposal to use the loadings or correlations of the original variables on the canonical variates. However, it would be advantageous to arrive at an understanding of the canonical weights which are comparable to the meaning of the regression coefficients in multiple regression.

MULTIPLE SET CANONICAL CORRELATION

The usual two-set canonical correlation analysis can be extended to three or more sets of variables. Although only one marketing application has so far been made (Lutz and Howard 1971) some discussion of the problems involved seems justified here because there are several marketing problems for which the multiple set canonical analysis seems most appropriate.

As in the Greek drama, the introduction of a third protagonist complicates the problem considerably. It is no longer obvious which of the three possible pairwise canonical correlations should be maximized, or whether a weighted combination of the three should be considered. The assumption of one dependent or criterion set and two predictor sets (or vice versa) does not clear matters up either, since we still have to decide which of two sets should be given most weight, or whether they should be considered equally important. As a consequence, the common distinction between analysis of dependence versus analysis of interdependence is no longer very relevant. Even if we impose the requirement that the multiple set case should be reducible to the two set case, no unique approach emerges. In fact, one of the most recent authoritative statements of the multiple set case considers no less than five alternative solutions which all reduce to the standard solution for two sets (Kettenring, 1971). Given this, there should be little surprise that the solution is often picked on the basis of computational efficiency rather than analytical objectives.

Formal Statement of Problem

Write the m sets of variables under study as $X_j' = (X_{j1}, X_{j2}, \ldots, X_{jp_j})$, j=1, ..., m, with $p_1 \leq \ldots \leq p_m$ so that the sets are numbered by increasing size. The joint distribution of the m sets is assumed non-singular. The problem is to find the m linear compounds (one for each X_j') which have a maximum correlation as defined below. For these compounds we write

$$Z_j^{(s)\prime} = \{Z_1^{(s)}, Z_2^{(s)}, \ldots, Z_m^{(s)}\}$$

where the superscript denotes the stage s=1, 2, ..., p. The $Z_j^{(s)}$, j=1, ... m are standardized with a variance of 1. The correlations to be maximized can be written as follows

$$\Phi^s = \begin{bmatrix} 1.0 & r_{Z_1^{(s)}Z_2^{(s)}} & \cdots & r_{Z_1^{(s)}Z_m^{(s)}} \\ r_{Z_2^{(s)}Z_1^{(s)}} & 1.0 & & \vdots \\ \vdots & & \ddots & \vdots \\ r_{Z_m^{(s)}Z_1^{(s)}} & \cdots & & 1.0 \end{bmatrix}$$

The five alternative solutions to the multiple-set canonical correlations can be directly formulated in terms of the correlation matrix $\Phi^{(s)}$ and its associated eigenvalues $\lambda(\Phi^{(s)})$. Basically, the solutions are as follows.

1. Maximize the sum of the correlations (SUMCOR) of $\Phi(s)$.
2. Maximize the variance (MAXVAR) i.e. maximize the largest $\lambda(\Phi^{(s)})$.
3. Maximize the sum of squared correlations (SSQCOR) of $\Phi^{(s)}$.
4. Minimize the variance (MINVAR) i.e. minimize the smallest $\lambda(\Phi^{(s)})$.
5. Maximize the generalized variance (GENVAR) of $Z^{(s)}$,

$$|\Phi^{(s)}| = \prod_{j=1}^{m} \lambda_j(\Phi^{(s)}).$$

These methods have in common the feature that they all reduce to Hotelling's classical procedure when there are only two sets. In addition, they all need only the correlation matrix as the basic input.

All five methods yield comparable first stage canonical variates in that the associated correlation matrix $\Phi^{(1)}$ is equal to the identity matrix if the m sets are mutually uncorrelated. Additionally, the values of the m sets in different stages are uncorrelated. The procedures generate those canonical variates which, in somewhat different ways as we have seen, are as far from the no-correlation case as possible.

One would like to select the particular procedure on the basis of the data at hand and the objectives of the analysis, but at this stage not enough empirical research has been generated to provide simple and directly applicable guidelines. In general, the MAXVAR or MINVAR solutions seem preferable since there exist analytical solutions for them and there is no need for iterative algorithms. An additional advantage of these two methods is that they generate information on which two of the sets possess the largest first canonical correlation. Whenever the other methods are computationally feasible, however, the prospective user would do well to use more than one of the methods proposed in order to develop an understanding of the sensitivity of the solutions to the method employed.

APPLICATIONS OF CANONICAL CORRELATION IN MARKETING

The following review of applications of canonical analysis in marketing research will focus primarily upon the methodological aspects of the research and only secondarily upon the substantive findings. Although much has been written on canonical correlations, and the technique has been used extensively in the behavioral sciences, the published applications in marketing are surprisingly few. The scarcity of canonical analysis applications is probably directly traceable to the difficulty of interpretation.

One of the first published articles on canonical correlation in marketing research was Green, Halbert, and Robinson's expository paper (1966). The empirical application they presented mainly served illustrative purposes but is still of some interest here. The aim of the analysis was to relate individual game playing behavior to personality traits. They used two criterion variables, "sensitivity" and "bias," reflecting the mode of behavior observed in an experimental business gaming situation. The personality variables consisted of scores on five tests, all self-administered. The subjects were 36 graduate business students. Only the first canonical coefficients are presented and discussed by the authors although both roots were significant at the .05 level-- an unfortunate approach considering the expository nature of the paper. Only scant attention was given to the substantive interpretation task, with the consequence that the reader gets a somewhat superficial picture of the real challenges involved in canonical correlation. On the other hand, one might say that we have today come a long way further, and that in 1966 the article fulfilled a necessary mission.

An ingenious application of canonical correlation was made by Kernan (1968). He attempted to relate decision behavior and personality traits. Faced with no significance of the relationships on a variable-to-variable basis, he turned to canonical correlation in order to see whether or not a linear combination of decision behavior acts could not be seen as functionally determined by a combination of traits. Identifying no less than 32 different acts as describing the criterion set and eight different personality traits as the predictor set (the presence of more criterion than predictor variables requires that the analysis be run in reverse), he found that the first canonical root was significant only at the .10 level. He concluded that while there is at least some connection between personality and decision behavior, it needs to be verified and perhaps amplified by other multivariate techniques such as cluster analysis. The clustering used the subjects' scores on the criterion variables to develop a hierarchical ordering of the number of clusters possible. Using a within-sum-of-squares criterion, the optimal clustering level was judged to be the one where four clusters of subjects obtained. Kernan then showed how the personalities of the people thus classified differed across clusters.

One drawback of Kernan's work from our vantage point is that he apparently did not utilize the canonical weights at all in the clustering approach. As we have seen, there are reasons for being careful when weights are interpreted. On the other hand, the canonical weights ought to have been used to scale the original criterion variable scores of each subject. In this way those criterion variables seen as important in the canonical analysis would have been given a higher weight in deriving the similarity indices between the subjects in the clustering phase of the analysis. Such a use of the results from the canonical analysis would have been much more logical than assigning each variable equal weight. As the article now stands, the canonical correlation was only used as a justification for going on into the clustering approach; not much substantive information from the canonical analysis was utilized in the clustering stage.

In a study drawing upon Kernan's analysis, Sparks and Tucker (1971) also used canonical correlation to relate personality traits and product usage. They related 17 criterion variables on product use to the same 8 predictor variables used by Kernan. In contrast to Kernan, they obtained three significant canonical roots. Accordingly, the simple clustering scheme used by Kernan was not utilized to cluster subjects. Instead, Sparks and Tucker attempted to interpret the canonical results, relying on the weights that were higher than .30. Although some traits were strong in more than one canonical relationship, they were able to interpret most of their findings on the basis of personality theory. In addition, some parallels to Kernan's clustering approach were drawn on the basis of six clusters, but generated no more insights than the canonical results had already provided.

The results-- particularly the fact that three significant canonical relationships indicate interactions between personality variables-- led the authors to hypothesize that no general mode of consumer behavior is possible (Sparks and Tucker, 1971, p. 70). It should be clear that such a conclusion is too far reaching considering their analytical framework, with only a few personality traits incorporated into the consumer behavior model. Rather, the more modest hypothesis that the "trait interactions or nonlinear relationships may compose a significant portion of the personality-behavior relation" seems more realistic.

In a paper evaluating the linkage between degree of socio-economic risk and interpersonal influence, Perry and Hamm (1969) used canonical correlation for the statistical test of the derived relationship. The data came from a questionnaire administered to 101 male undergraduates on the degree of risk and interpersonal influence for 25 products.

The questionnaire had two sections. In the first section the respondents were asked to rate, separately, the significance of social risk and that of economic risk for each decision. In the second section several information sources for each product were evaluated by the respondents. Only two interpersonal sources (observing others, and verbal communication from others) were used in the final analysis, however. Twenty-five canonical analyses were run (one for each product) with the two risk variables as predictors and the two interpersonal sources as the criterion variables. Ten of the first stage canonical roots were significant at the .05 level. In addition to the test of the canonical correlations, the authors also used Spearman's rank-order correlation to test the significance of the relationship between risk and interpersonal influence, and found it significant at the .01 level.

No mention was made in the article of the second canonical roots and their significance. Clearly, for the authors' purpose, the significance test of the first canonical root was sufficient. On the other hand, one would have liked to see a bit more detailed analysis of exactly what the canonical results were. In addition, the non-significant roots were ascribed to nonlinearities, which were accused of yielding, "insignificant correlation whenever the actual correlation is nonlinear" (p. 171). More important, the authors made no attempt to specify what the nonlinearities might be, and then to try to evaluate their impact on the system of correlations. Although the article represents overall a very appropriate use of canonical correlation, it also seems to fall short of exploiting the full potential of the approach.

In a recent expository article, which partly served as a follow-up on the earlier Green, Halbert, and Robinson article (1966), Alpert and Peterson (1972) approached the interpretive problems of canonical correlation. They discussed the various measures to use when judging the amount of explanation offered by the canonical variates, with an emphasis upon the redundancy measure discussed earlier in this paper. Less is offered on the problem of the interpretation of the weights associated with the canonical solution, although an interesting comparison between the weights and the loadings (simple correlations) of the variables on the canonical variates is made. Their recommendation is that in addition to weights, the loadings and also the proportion of squared loadings attributable to the variable in that particular canonical variate be used. The latter index might show that a variable is an important determinant in a late, but significant, canonical variate, although its weight and loading is higher in a previous variate.

The empirical illustration used by the authors related four product use variables (the criterion set) to five demographic indices (the predictor set). The authors provided little substantive explanation of their results since their purpose was different, but according to the usual statistics, the demographic variables did have a significant predictive power with respect to product use. The authors do not attribute very much to the fact that three of the possible four canonical roots are significant at the .02 level, however. They point out that according to the redundancy measure (see Table 1) only about 17% of the total variance is shared between the two sets of variables.

Table 1. Components of Redundancy Measure in the Alpert and Peterson Study (1972)

RELATIONSHIP	CANONICAL R	R^2(or λ)	VARIANCE EXTRACTED, VP or VC	REDUNDANCY ($\bar{R}^2$), $\lambda \cdot$VP, OR $\lambda \cdot$VC	PROPORTION OF TOTAL REDUNDANCY
Predictor Set					
1	.6180[a]	.3819	.2999	.1146	.6753
2	.4104[a]	.1684	.2481	.0418	.2463
3	.2510[a]	.0630	.2082	.0131	.0772
4	.0458	.0021	.0856	.0002	.0012
				.1697	
Criterion Set					
1	.6180[a]	.3819	.2994	.1143	.6978
2	.4104[a]	.1684	.2174	.0366	.2234
3	.2510[a]	.0630	.1451	.0123	.0751
4	.0458	.0021	.2881	.0006	.0036
				.1638	

[a] $p < .02$.

Against the limited interpretation offered by the authors, one might argue that in fact the canonical solution gives the particular nature of the relationship between demographics and the product usage desired. While it should be kept in mind that not that much variation seems to be explained, one might still try to interpret this pattern as more than just noise, given that the significance test is passed.

The authors recommend finally that canonical analysis should be treated as an integral part of a large battery of exploratory techniques, to be abetted by other methods when the data exploration has yielded testable hypotheses. There is very little in the paper that attempts to show how such further analysis would be integrated into the canonical approach.

There have also been a few applications of canonical correlation to the attitude-behavior relationship. Sheth (1971b) developed several dimensions of both attitudes and behavior, and used canonical correlation to

predict the set of behavior variables from the attitude measures. Three criterion variables (purchase of a given brand, intention to buy, and liking of the brand) were correlated with 7 evaluative beliefs in one run, and to evaluative beliefs plus 12 social and situational factors in another run. Thus, canonical correlation was partly used to test whether these last factors would provide any additional predictive power beside the evaluative beliefs with respect to different behaviors.

The results of the canonical analyses were very much the same with two dimensions significant (at the .05 level). Basically, one pair depicted the liking dimension, the other the purchase and intention dimensions. The liking dimension was mainly explained by the evaluative beliefs variables, whereas the introduction of the social and situational factors served to improve significantly the explanation of the purchase and intention dimensions. One basic finding was that evaluative beliefs related to liking were different from those related to purchase and intention.

Because the relationship between attitude and behavior has been investigated quite intensely in recent years, there exist some very tight conceptualizations for the relations. In his paper, the author was able to draw upon these past conceptualizations and to develop them further into a testable model stage. One would have hoped that some aspects of the applicability of linear weighting schemes would have been discussed; and that, following the results, some attention to the possibility of nonlinearities should have been given. Overall, however, the approach as used here generates a surprising degree of insight into relatively uncharted terrain, and it remains for future hypothesis-testing research to establish whether some of the findings in fact can be shown to hold more generally.

Another quite different area of application of canonical correlation to marketing problems deals with the relationship between prices of products (or brands) and the quantities bought of those products. It is, in other words, a type of demand analysis, and the first applications in this area occurred in economics in the 1940's (see, for example, Waugh, 1942). For competing products it is quite natural to use the canonical correlation as a method of analysis since the weights tend to indicate where high and low price cross elasticities occur. The interpretation problems, compounded by the so-called simultaneous equation problem (where prices and quantities tend to be set as a result of interactive rather than undirectional forces) have led to little use of canonical correlations in economic demand analysis, however.

In marketing the prevalence of panel data in which brand purchases and prices are immediately available and where the simultaneity problem is mostly nonexistent (because of the high frequency of purchase and the imperfectly competitive situation), the canonical approach seems more promising. The Carmone paper included in this volume represents one of the first marketing applications in this area. Since his paper is extensively reviewed by Carman's discussion, we will not elaborate more in this paper. It should be pointed out, however, that Carmone's interpretation of the second pair of canonical variates might be questioned. The reason is that canonical correlation-- as we have seen-- yields only as many roots as the number of criterion variables (here two). As some reflection will show, the last root will consist of the eigenvalue for which the relationship between criteria and predictors is at a minimum. This is because the maximization procedure followed in locating the canonical variates is identical to what a minimization procedure would be. Thus, if for some reason one wanted to minimize the canonical correlation, the smallest eigenvalue and its related eigenvectors would be chosen first. In two-set analysis, accordingly, one would do well not to attach too much importance to the last eigenvalue other than as a "lower bound."

In a recent article on market segmentation, Frank and Strain (1972) used canonical correlation in a new and interesting fashion. They identified two dependent variables, measuring the quantities bought of two product types of a frequently purchased grocery item. The observations were generated from housewives' panel diaries over a 25-week period. At the end of this period a quiz was administered to each housewife and information was collected on attitudes, interests, and opinions, as well as more general measures on life style and personality (an AIO-battery). These measures constituted the independent variables.

After a factor analysis with a varimax rotation had reduced the number of AIO-measures to 26, a canonical correlation analysis was run. As could be expected on the basis of the large number of independent variables, both canonical roots were significant above the .05 level. Since both product types loaded heavily and with the same sign on the first y-variate, the authors concluded that this represented "primary" demand for the product. The second y-variate showed opposite signs for the two dependent variables, interpreted as representing "selective" demand.

The authors then proceeded to interpret the canonical weights of the different explanatory variables in order to generate some insight into the determinants of primary and selective demand. They found basically that primary demand was determined by the degree to which the housewife had a negative or positive outlook on life, whereas selective demand (the second variate) was determined by the stage of life cycle. Then the authors pointed out: "Studies using canonical correlation analysis usually end their report of results at this point." In this case, however, the analysis using the canonical results continued.

To derive some indication as to the number, nature, and relative size of the customer segments representing different combinations of these personal characteristics, the authors partitioned the respondents into segments based on the results of the canonical analysis, as follows. First, the values of the two canonical variates from the independent side were computed for each individual. Then, these values were broken into three categories, high, medium, and low, and the respondents were cross-classified according to their scores. In this way five specific segments (out of a possible nine) were explicitly identified: Young Independents, Young Dependents, Old High Achievers, Old Low Achievers, and Middle of the Road, basically reflecting the positive/negative outlook (from the first variate), and the stage of life cycle (the second variate). The authors then identified the specific characteristics of the segment members, and analyzed the actual purchase behavior (in terms of amounts bought of each type of product) of the different segments. Of minor interest in this context, the authors also derived implications for product designs and basic appeals, although little attention was given to the identifiability and reachability of the different segments.

It should be clear that the type of extended analysis of canonical correlation results which is undertaken in the paper represents a good example of what can be done with results from canonical analysis in marketing. The initial results are utilized for further analysis of the market segmentation problem studied. At the same time, a few questions might be raised. First, it is not clear why the three-level cutoff was chosen and why the authors avoided the more common clustering approach after the dimensions (the two independent variates) had been identified. Also, the use of the second root in the two-dimensional case is questionable, since (as noted earlier) the last root will represent the minimum-correlation pair of variates. Third, one would have liked some statistical test of the membership-in-the-segment figures, especially from the cross-tabulation, indicating to what extent differences were due to the

first or second variate. This would have enabled the reader to make a better evaluation of the possible strengths of the findings outside the particular application, and the relative segmenting power of the first and second variates.

Overall, however, the article represents a solid step forward in the application of canonical correlation and the use of its results.

In an effort to develop an understanding of the competitive structure of the margarine market, Johansson and Sheth (1973a) used canonical correlation relating weekly average market prices and quantities of different brands. The ten largest brands in the market were used. After the usual canonical analysis had been run, the significant (at the .05 level) variates were rotated orthogonally (using Varimax) before interpretation. As a consequence, the interpretation of the findings became much easier--although it is somewhat questionable whether such a rotation is statistically justifiable.

The original and rotated solutions are displayed in Tables 2 and 3. The first five canonical variates are significant (at the .05 level) and were the ones rotated. The substantive results point to some brands as being substitutes and thus in competition (where the cross elasticities--the weights-- are positive and high), others are complementary (where the weights are negative) and some brands simply occupy their own competitive "niche" (failing to appear strongly on any of the significant variates). A particularly interesting result is the non-reciprocal cross elasticity uncovered between brands 3 and 10: Price changes for brand 3 affect brand 10's sales, but not vice versa. Clearly, these findings should be seen as tentative, to be tested by further research, but they do suggest the need for a modification in the existing economic thinking about competition.

Again, the usual deficiencies of the canonical analysis can be found here. Nonlinearities were not explicitly considered, even though most demand functions are modeled nonlinearly. However, over the range covered by the data, a linear relationship seemed to hold quite well. Another drawback is that no factors besides price were considered. It is clear that in most cases price is only a very minor determinant of the quantity sold. On the other hand, in line with some of the results from behavioral research, one might argue that the analysis uncovered at least the basic pattern of relationships between prices and quantities sold. If price is strongly correlated with some other explanatory variables not included (such as advertising and distribution coverage) there is of course the chance of a spurious pattern. Overall, however, the study should be seen as largely exploratory, with the hypotheses emerging to be tested through confirmatory analysis.

An interesting application of canonical correlation was made by Farley and Ring (1974) who attempted to re-specify the Howard-Sheth (1969) buyer behavior model inductively from panel data. They identified 11 criterion and 17 predictor variables which had originally been used to specify one fairly complete version of the Howard-Sheth model as a system of equations (Farley and Ring, 1970). Since this earlier test of the model had been largely unsuccessful, the authors were led to backtrack and attempted to respecify the relationships on the basis of canonical correlation analysis and the Automatic Interaction Detector (AID).

The results seemed to indicate that the correct model (as far as can be judged from these data) contained a greater array of feedback loops than originally conceptualized. Also, indications were that a small number of predictors explained variations in many of the criterion variables, and should be introduced as direct determinants of some of the criterion variables conceptualized at a relatively late stage in the original model. There were five significant canonical roots, and a fairly consistent set of weights associated with these variates, making for only minor difficulties in interpretation. Eighteen of the original relationships were confirmed, thirteen were not confirmed, and several new causal paths were suggested.

Table 2. Canonical Analysis of Price-Quantity Variables in the Johansson-Sheth Study (1973a)

FUNCTION	EIGENVALUE	CORRELATION	WILKS LAMBDA	CHI-SQUARE	DF
1	0.7900	0.888	0.0049	220.4383	100
2	0.6524	0.8077	0.0235	115.6742	81
3	0.6120	0.7823	0.0676	111.8266	64
4	0.4084	0.6391	0.1741	72.5402	49
5	0.3407	0.5837	0.2944	50.7528	36
6	0.3292	0.5737	0.4465	33.4640	25
7	0.2521	0.5021	0.6656	16.8953	16
8	0.0691	0.2628	0.8899	4.8418	9
9	0.0405	0.2012	0.9559	1.8724	4
10	0.0038	0.0614	0.9962	0.1566	1

Matrix* of Criterion Weights

	1	2	3	4	5
Quantity, Brand 1	-0.39759	0.17341	-0.03855	0.47071	0.37221
Quantity, Brand 2	0.20604	-0.32625	0.44315	0.59724	0.43645
Quantity, Brand 3	0.08795	0.20906	-0.42405	0.00744	-0.06724
Quantity, Brand 4	-0.25863	0.77067	0.46134	-0.01939	0.16902
Quantity, Brand 5	0.49386	0.47838	-0.19002	-0.12566	-0.27487
Quantity, Brand 6	-0.20884	0.08510	0.12558	0.39249	-0.11814
Quantity, Brand 7	0.17414	-0.40619	0.36654	0.09703	0.29456
Quantity, Brand 8	0.08239	-0.02196	0.18659	-0.51717	-0.18296
Quantity, Brand 9	0.27960	0.00778	-0.14166	0.50750	-0.44964
Quantity, Brand 10	-0.50981	0.06025	0.43321	0.03530	-0.59927

Matrix* of Predictor Weights

	1	2	3	4	5
Price, Brand 1	0.61124	-0.46582	0.32033	-0.24499	0.07676
Price, Brand 2	-0.17010	0.07430	-0.11964	-0.38689	-0.35252
Price, Brand 3	-0.37325	0.11712	0.56693	0.30895	0.01395
Price, Brand 4	-0.07633	-0.64315	-0.33481	0.02428	-0.68219
Price, Brand 5	-0.47317	-0.40638	0.25968	0.17731	0.71618
Price, Brand 6	0.12580	0.06679	-0.02275	0.16921	0.14934
Price, Brand 7	0.17004	0.23673	-0.28224	0.34104	-0.31225
Price, Brand 8	0.14327	-0.06134	-0.33832	0.51152	0.31017
Price, Brand 9	-0.14592	0.38864	-0.25229	-0.35827	0.02776
Price, Brand 10	0.18318	0.03435	-0.42572	-0.06690	0.42425

*Only significant canonical variates are included using 0.05 significance level.

As to the appropriateness of the canonical analysis here, one would have to question somewhat the reason behind such a "quantum jump" from a complete specification, as in the 1970 article, to the present empirical inductive approach. The earlier results must contain some information which could have been utilized for the respecification rather than starting from scratch, especially since the same data base was used. Furthermore, even assuming that the authors wanted to start anew, the inductive process could clearly have been pursued in much greater detail, approaching the more specified formulations gradually.

Table 3. Rotated Canonical Space of Competitive Market Structure in the Johansson-Sheth Study (1973a)*

	ROTATED CANONICAL VARIATE I	II	III	IV	V	R^2
Criterion Set (Quantity)						
Brand 1	.68				.89	.46
Brand 2						.44
Brand 3						.27
Brand 4			.88			.57
Brand 5				.66		.58
Brand 6						.13
Brand 7					.44	.29
Brand 8	-.54					.37
Brand 9				.61		.38
Brand 10		.86				.41
Predictor Set (Price)						
Brand 1	-.65				.51	
Brand 2					-.50	
Brand 3		.59				
Brand 4			.94			
Brand 5				-.86		
Brand 6						
Brand 7				.56		
Brand 8	.49					
Brand 9					-.50	
Brand 10		-.62				

Values less than .40 omitted from the table.

Within their limited approach, the authors tend to overlook the great importance of the "Prior purchase" variable as a determinant of the criterion variables since it alone seemed to account for all the variation explained by the first variate. Not mentioned by the authors, also is the fact that most of the criterion variables (six out of eleven) have higher weights in the first variate as compared to the other four significant ones. Finally, the remaining five criterion variables have their second highest weights in the first variate. All in all, the first pair of variates seems much more important than any of the other four, and it is a pity that no partialled out redundancy index is included in the results. The redundancy attributable to the first pair is probably very high relative to the other pairs. The surprising clarity of the five significant pairs of variates does generate some interesting hypotheses that seem worthy of further testing, however.

Stanton and Lowenhar (1974) used canonical correlation to establish a linkage between individuals' needs and attributes of TV shows. The basic research hypothesis emanated from economic psychology where it has been argued that to be successful a firm's product or service offering must exhibit characteristics which the individual customer/consumer sees as needfulfilling. The authors used six attributes of the shows as the criterion set and the psychological need variables as the predictor set. Three significant canonical roots were derived. Basically, the authors saw a confirmation of the basic hypothesis in the fact that three variates were significant, and they indicated what the determining variables in each variate were based upon the weights. They also presented the redundancy measure partialled out for each pair of variates, but did not mention the fact that the redundancy attributable to the third pair was less (on the criterion side) than that attributable to the fourth (and insignificant) pair.

The space limitations imposed on the authors probably prohibited a more extensive presentation and discussion of the canonical correlation results, but also make it very difficult to ascertain whether the offered interpretation is justified. The full set of coefficients is not presented, but only those judged important in each variate. Similarly, no attempt is made to provide any specific directions for further testing of the uncovered empirical relations. Overall, the approach seems reasonable, however, and should provide interesting hypotheses for further confirmatory research.

One investigation into the attitude-behavior relationship carried out by Lutz and Howard (1971) represents the only application of multiple set canonical analysis so far to marketing problems. The authors investigated the relationship between four sets of variables using in part the same conceptualization and data as Sheth's (1971b) paper. The four sets consisted of belief-importance scores, beliefs, brand preference measures, and purchase measures. Three loosely stated hypotheses regarding the interrelationships between these sets of variables were tested using an original Horst (1965) iterative algorithm for the estimation of the canonical correlations. The procedure used amounts to a SUMCOR approach as discussed earlier in this paper. Since the correlations in the second set were small relative to the first, the last two canonical correlations were not computed.

The iterative procedure performed satisfactorily, converging quickly. The authors apparently did not perform any analysis of the effect of different starting points upon the final correlations, but seemed to be satisfied that optimal estimates had in fact been derived.

Generally, the authors hypothesized that the preference set would be more strongly related to the two belief sets than the purchase set. This hypothesis was confirmed. On the other hand, the hypothesis that belief-importance measures correlate more strongly with preference than simple belief measures was not confirmed, nor was the hypothesis that preferences would correlate higher with purchase variables than with either of the belief measure sets. These results generally held also for the second canonical correlations. Even though the statistical significance tests of the two-set canonical correlation do not carry over into the multiple-set case, the interpretation of the relative size of the coefficients appears warranted. Perhaps the most interesting finding is that the choice of belief-importance scores versus simple belief scores seemed to matter less for structural purposes than the choice of the attributes: the same attributes were weighted heavily in both variable sets.

As the authors emphasized, the multiple set technique should be seen as an exploratory data analysis technique, much as the case for the two-set theory. Thus, for example, one could visualize a second stage analysis in which a two-set canonical correlation would be run with belief variables against preference variables, and then progress into a simultaneous systems approach proposed earlier. It should be made clear, however, that the paper represents a very promising first excursion of marketing researchers into multiple set analysis.

CONCLUSION

In order to evaluate the appropriateness of the canonical correlation approach as compared to other approaches when attacking a specific problem there are certain factors to keep in mind. First, the canonical correlation approach basically assumes that the only *a priori* information the

analyst posesses with respect to the sets of variables is that they are distinct-- the particular structural way in which one set might relate to another is not known initially. It should come as no surprise that in vie of this lack of theoretical prior information, the canonical correlation approach consists of fitting simple linear combinations of the original variables in such a way that the correlations between the sets are maximized.

If some prior additional information is available, the researcher might still want to use the canonical analysis-- especially where the aim of the analysis is the reduction of the data to a manageable few dimensions When the aim is for structural interpretation of the estimated coefficients however, the canonical correlation approach should be replaced by other approaches, which incorporate the additional information. One reason is that the estimates tend to be more efficient since fewer parameters are estimated. Another reason is the fact that the weight estimates generated by canonical correlation tend to be very difficult to interpret. If more information is built into the estimation procedure, interpretation generally becomes easier.

The difficulty of interpretation has probably been the main factor in the relatively low number of published articles in marketing research which use canonical correlation analysis. As we have seen, the main use of the technique has been in exploratory and descriptive data analysis, where the relationships between fairly large numbers of variables can sometimes be shown to be reducible to just a few basic dimensions. As soon as the initial canonical analysis yields some understanding of the dimensionality and basic nature of the relationships, however, techniques which are more dependent upon prior specifications should be utilized to augment canonical correlation analysis. These latter techniques are generally preferable for testing the uncovered relationships even if ingenious use of canonical analysis in testing is possible.

NOTES

1. We will skip illustrative empirical example of canonical correlation analysis in order to conserve space. The reader is urged to follow an excellent example developed by Frank Carmone in the preceding chapter.
2. This claim was first made, but not proven, by Stewart and Love (1968), and was disputed by Nicewander and Wood (1974). A conclusive proof is possible, however--see Johansson and Lewis (1974).

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