FACULTY WORKING PAPERS

College of Commerce and Business Administration

University of Illinois at Urbana-Champaign

October 13, 1976

A COMPARATIVE ANALYSIS OF MULTIDIMENSIONAL SCALING OF PORTFOLIO INVESTMENT

Gary A. Anderson and Jagdish N. Sheth

#344

COMPARATIVE ANALYSIS OF A COMPLEX DATA SET

ABSTRACT

This paper examines two individual differences multidimensional scaling models in assessing the structure underlying institutional investors' perception of common stock investment. Comparison of the 3-Mode scaling model with the INDSCAL model confirmed the applicability of the more easily interpretted INDSCAL solution. Examination via both models also provided useful information in assessing the dimensionality of the stock space and the person space. Results indicate that the investors studied (corporate pension, personal trust, commingled, and insurance fund portfolio managers) compared the stocks along the following three independent dimensions: situational risk, variability, and return. The models reported nearly identical (within a transformation) perceptual configurations. Comparison of the person spaces disclosed dependence among the INDSCAL weight dimensions, i.e., the INDSCAL weight matrix contained only one dominant dimension.

INTRODUCTION

The purpose of this study is to compare two different individual differences multidimensional scaling models in assessing the structure underlying a group of institutional portfolio managers' perception of common stock investment. Specifically, the objective of the paper is to cross-validate the results of the well-known INDSCAL (Carroll and Chang, 1970) with respect to both object (stock) space dimensions as well as person (investors) space dimensions with 3-Mode factor analysis (proposed by Tucker, 1972).

Gary A. Anderson
Ohio State University

Jagdish N. Sheth, Professor
Department of Business Admin.
University of Illinois

In multivariate analysis of complex realities, it is becoming increasingly important to ensure that the results of analysis are not subject to statistical artifacts. In bringing about getting the most out of multivariate methods, Sheth (1975) makes the following observation:

> "Guard yourself against the danger of making substantive inferences about market realities which may be an artifact solely due to the peculiarities of a particular multivariate method. Since multivariate methods are more complex statistical procedures, there are many more underlying assumptions required for their optimization (minimization or maximization) and it is easier to inject substantive meanings in the data even if the data are essentially random relationships. This has been especially true of those multivariate methods such as cluster analysis, multidimensional scaling and conjoint measurements which possess no underlying sampling theory, and therefore, are essentially heuristics often no better than naive judgmental rules." (pp. 15-16)

In order to guard against this danger, Sheth has recommended that the same data be subjected to at least two different techniques, or at least two variations of the same basic multivariate method. The present study is therefore, essentially a cross-validation of INDSCAL mapping results by another technique, namely 3-Mode factor analysis.

As a multidimensional scaling model, INDSCAL has many nice features and advantages which makes it a very tempting technique. First, the method takes into account individual differences in the mapping of stimuli. Second, the method allows for projecting of attribute information into the space derived from the similarity judgments. This enables the researcher to identify the dimensions in a more objective manner. Third, the results of INDSCAL are easy to understand and to communicate to others especially those not equipped with the knowledge of multivariate methods. Given these advantages, it is easier to ignore or to at least minimize the underlying assumptions of INDSCAL resulting in the danger alluded to by Sheth. Accordingly, the INDSCAL results should be subjected to cross-validation procedures.

DESIGN OF THE STUDY

The portfolio managers' responses to a self-administered questionnaire comprise the primary data base. In the questionnaire the following categories of data were collected for the 15 stock stimulus set:

1. similarity/dissimilarity data;
2. reliability data;
3. respondent reported criteria for making similarity judgments; and
4. attribute rating data.

The 15 stocks included in the questionnaire, over which the categories of data were collected, are:

1. International Business Machines;
2. American Telephone and Telegraph;
3. Eastman Kodak;
4. Sears Roebuck and Co.;
5. Polaroid;
6. Avon Products;
7. Aetna Life and Casualty Insurance;
8. National Airlines;
9. Exxon (Standard Oil of New Jersey);
10. National Steel;
11. Beatrice Foods;
12. St. Regis Paper;
13. General Motors;
14. Consolidated Edison of New York; and
15. LTV Corporation.

Ratings for each of the 15 stocks were collected on the following attributes:

1. liquidity;
2. expected percentage return from investment over the next three years;

3. perceived risk of investment;
4. current year's expected dividend yield;
5. active trading profit potential over the next three years;
6. consistency of growth in earnings per share over the last five years;
7. outlook for the firm compared to other firms within the same industry;
8. management of the firm;
9. potential of market for product and services from the firms;
10. price appreciation potential over the next three years;
11. perceived credit risk of each firm;
12. sensitivity of price to interest rate changes;
13. influence government has on each firm;
14. business risk;
15. predictability of earnings over the next three years;
16. possibility of loss;
17. volatility of stock price over the last five years;
18. variation of expected returns; and
19. outlook for industry stock performance.

The first section of the questionnaire, following the cover letter, consisted of an instruction sheet which explained the nature of the task and familiarized the participant with the set of 15 stocks.

The second section of the questionnaire consisted of the 105 possible pairs of the 15 stocks (arranged in a Ross, 1934, ordering)[1] and a set of 15 repeat pairs (the initial 15 pairs in reverse order). The respondent's task was to rate the similarity/dissimilarity of each of the 120 pairs of stocks on a nine-point scale ranging from 1 (very similar) to 9 (very dissimilar). The first and second categories of data were thus collected in this second section of the questionnaire.

Immediately following the similarity questions, the third section of the questionnaire elicited the remaining categories of data: (a) the portfolio

[1]Ross ordering is a statistically randomizing procedure for arranging the possible pairs of stocks to avoid bias towards any particular stock.

manager's self-assessment of the criteria he/she used in making the similarity judgments; and (b) the portfolio manager's ratings of each of the 15 stocks over the above listed attributes.

When it became apparent from a pilot test of the questionnaires that phone contact or contact by mail did not create sufficient interest to generate a favorable response rate, personal interviews with supervisors of portfolio managers were arranged. Since interviews with these supervisors were necessary (with the exception of those insurance companies where previous close contacts existed) the proposed sampling from many geographical areas was abandoned. Sampling from a diverse geographical group would have slowed data collection and introduced effects due to a changing market as well as true perceptual variation. As a result, the seven banks included in the study are all from the Chicago area, and the nine insurance companies represent a sampling from companies where prior contacts with the supervisor or portfolio manager of the insurance company existed.

The collection period extended from May 3 to July 18, 1974 with the preponderance of questionnaires being completed in the period June 20 to July 10. The questionnaires completed in May represented responses to the pilot test. Since no difficulties were raised in the pilot test, they were incorporated into the main study.

To minimize the effects of a changing market it was desirable to have all questionnaires completed on the same date. Although this was not achieved, the majority of the questionnaires were completed within a relatively short period of time in which the market was inactive.

Stocks were selected such that they would 1) elicit variable weighting for the 19 respondent-rated attributes such as expected return, potential for price appreciation, perceived riskiness, etc., and 2) exhibit variable ratings across available objective attributes such as P/E ratio, dividend yield, and beta. In addition, stocks were selected from various industries, and stocks were selected such that institutional favorites (those held in high concentration by most institutional managers) and disfavorites (those held in few if any institutional portfolios) were included.

METHOD OF ANALYSIS

Individual differences multidimensional scaling (INDSCAL) and 3-Mode scaling were employed in estimating the structure underlying the portfolio managers' comparisons of the stocks. The following brief summary of the general theory of multidimensional scaling (MDS) may clarify the design of the study and the method for recovering the underlying structure. For a detailed development and explanation of the MDS models compared and used in this paper, the reader is directed to Carroll and Chang (1970) and Tucker (1972).

The general MDS modeling may be summarized as follows:

$$\underset{rN}{X^T} \rightarrow \underset{NN}{D^T} \rightarrow\rightarrow \underset{NN}{\Delta} = \underset{NN}{D} \rightarrow\rightarrow \underset{NN}{\hat{D}} = F(\underset{rN}{X})$$

where r = the number of dimensions in the stimuli space;

N = the number of stimuli

$\underset{rN}{X^T}$ = the configuration of the respondent's true space, i.e., the structure and positioning of the stimuli in the respondent's perceptual framework.

$\underset{NN}{D^T}$ = the respondent's true configuration mapped onto a distance configuration;

Δ_{NN} = the matrix of similarities/dissimilarities between all possible pairs of stimuli in the individual's true distance configuration;

→ → the transformation is assumed to be montonic;

D_{NN} = the matrix of similarities/dissimilarities as reported by the individual in the questionnaire;

$\hat{D}_{NN}$ = the matrix of distances between pairs of stimuli derived from the multidimensional scaling of the reported similarities/dissimilarities;

$F(X_{rN})$ = the MDS function which defines distance in terms of the scale values, X, along the r dimensions for the N stimuli.

The following assumptions are implicit to the theory of MDS: 1) there exists a structure with a finite number of indices (r) through which the respondent (portfolio manager) relates the N stimuli (common stocks); 2) the respondent (manager) can accurately report the relationships among the stimuli (stocks) in his/her "true" perceptual space; and 3) the reported relationships (similarity/dissimilarity) can be accurately approximated in an r dimensional configuration. The basic premise of MDS is that the derived coordinates X_{rN} of the stimuli (stocks) are meaningful representations of the manner in which the respondent views the stimuli (stocks) along the r indices, i.e., the derived space accurately portrays the respondent's (manager's) "true" perceptual structure.

The range of multidimensional scaling models extends from fully metric to nonmetric and from group to individual space formulations. Since this research examines differences among and within subsamples of institutional investors, two different models from the class of individual differences models were employed. Inputs to both the INDSCAL (INdividual Differences SCALing by Carroll and Chang, 1970) and 3-Mode (Tucker, 1972) models were scalar products matrices formed from the respondent reported pairwise similarity judgments (Torgerson, 1958).

Selection of the INDSCAL model was based upon its wide utilization, ease of interpretation, and its history of accurately recovering perceptual structure. The INDSCAL model, unlike separate analysis of each individual, explicitly incorporates individual weightings of the dimensions in developing the group space. This allows not only differential structures (zero weighting by some, higher weighting by others) but also integrates the communality among the managers in developing the aggregate perceptual framework. The model is somewhat restrictive to the general theory of MDS, however, because it assumes 1) the relationship between similarity/dissimilarity and the derived distances is linear, and 2) the respondent's true perceptual configuration is orthogonal. The model is also powerful in that the solution meets a least squares-criterion and the solution is unique (rotations are not permitted) because the derived subject weights define a specific configuration.

The INDSCAL model is a special formulation of the more general 3-Mode model (see MacCallum, 1975). The 3-Mode analysis assumes that individual differences in responses are the result of not only differential weighting of the object dimensions but also differences in the perceived relations among the dimensions of the object space. The INDSCAL model assumes that the only source of observed individual variation is the weighting of the set of orthogonal object space dimensions. The differential relations among the dimensions of the object space in the 3-Mode model are represented by differential angles between pairs of dimensions. Off-diagonal entries in the core matrices for the person dimensions reflect the degree of obliqueness in perception. Since both models were employed, the comparison of the two solutions allows a direct examination of the simplifying assumptions of the INDSCAL model. In addition, the 3-Mode analysis provides alternative data for determining the appropriate dimensionality of the object and person spaces and for examining the possibility that the INDSCAL solution may be a local minimum. Analysis via both models hedges against the danger noted by Sheth (1975).

RESULTS

In addition to all possible pairwise comparisons of the 15 stocks (105, i.e., n(n-1)/2) 15 repeat pairs of stocks were included in the questionnaire. Simple correlations of the similarity/dissimilarity judgments of the 15 repeat pairs with the previous judgments indicated the managers' consistency in comparing the stocks (see Table 1).

Before examining the nature of the underlying considerations, the number of dimensions necessary to explain the perceptual relationships among the stocks must be determined. Each scaling model provides a measure of the recoverability, i.e., how well the derived space reproduces the participant's reported relationships among the stocks. For INDSCAL, the recovery measure is the correlation between the derived distances among the stimuli and the scalar product matrices formed from the reported similarity measures (see Table 2). The 3-Mode model reports the eigenvalues for each dimension (percent variance explained by each dimension equals the eigenvalue for the dimension divided by the sum of the eigenvalues).

Table 1: Individual Reliability and Recoverability Measures

Individual Number	ISCC	ISRC	Individual Number	ISCC	ISRC
1	0.857	0.831	28	0.780	0.828
2	0.773	0.783	29	0.748	0.479
3	0.784	0.634	30	0.725	0.565
4	0.782	0.912	31	0.708	0.413
5	0.565	0.671	32	0.808	0.928
6	0.728	0.802	33	0.765	0.969
7	0.728	0.781	34	0.658	0.816
8	0.772	0.678	35	0.732	0.296
9	0.795	0.707	36	0.837	0.882
10	0.645	0.647	37	0.555	0.357
11	0.762	0.687	38	0.789	0.554
12	0.828	0.777	39	0.736	0.756
13	0.727	0.489	40	0.621	0.983
14	0.783	0.776	41	0.637	0.844
15	0.626	0.775	42	0.820	0.747
16	0.687	0.922	43	0.626	0.569
17	0.789	0.702	44	0.906	0.912
18	0.722	0.945	45	0.626	0.252
19	0.695	0.811	46	0.795	0.788
20	0.653	1.000	47	0.800	0.724
21	0.669	0.734	48	0.815	0.859
22	0.794	0.846	49	0.840	0.966
23	0.780	0.776	50	0.802	0.965
24	0.753	0.663	51	0.810	0.789
25	0.623	0.124	52	0.709	0.577
26	0.872	0.805	53	0.747	0.457
27	0.830	0.934			
Mean	0.744	0.726	Std. Dev.	0.08	0.20

ISCC = Individual Subject Correlation Coefficient. This is the INDSCAL measure of recoverability for each individual. It represents the correlation between the respondent's scalar product matrix of the 105 pairwise comparisons and the 105 derived distance measures.

ISRC = Individual Subject Reliability Coefficient. This represents a measure of the consistency with which the manager completed the similarity portion of this questionnaire. The coefficient is the correlation between the respondent's first 15 pairwise judgments and these 15 judgments repeated at the end of the questionnaire.

Table 2

Variance Explained by Object Space Dimensions in
INDSCAL Stock Space

	Correlation	Cumulative Variance Explained
1-Dimensional Solution	0.57	0.34
2- " "	0.71	0.50
3- " "	0.75	0.56
4- " "	0.77	0.59
5- " "	0.79	0.62

Variance Explained by Object Space Dimensions in
3-Mode Stock Space

	Eigenvalue	Variance Explained	Cumulative Variance
Dimension 1	21.01	39.65	39.65
" 2	10.64	20.09	59.74
" 3	4.26	8.05	67.79
" 4	2.42	4.57	72.36
" 5	2.18	4.13	76.49
" 6	2.02	3.82	80.31

Variance Explained by Person Space Dimensions in
3-Mode Person Space

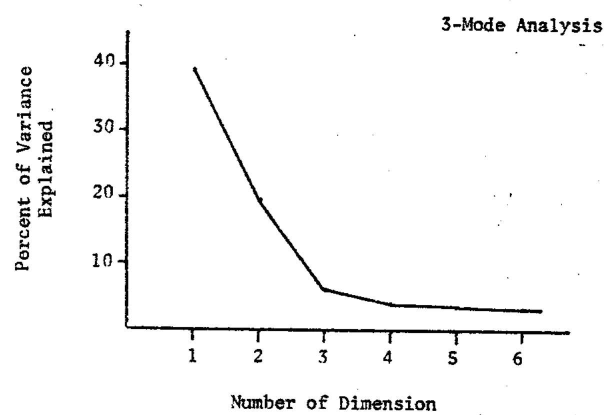
	Eigenvalue	Variance Explained	Cumulative Variance
Dimension 1	31.37	59.19	59.19
" 2	2.18	4.11	63.30
" 3	1.75	3.30	66.60
" 4	1.59	3.01	69.61
" 5	1.18	2.24	71.85
" 6	1.14	2.15	74.00

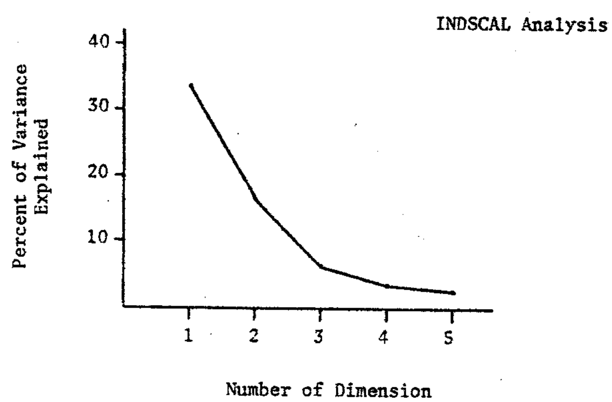
Plotting the measures of recoverability often reflects the "true" dimensionality of the perceptual space. One looks for a series of significant changes in the measure up to the "true" dimensionality followed by a series of very small changes (often linear) for dimensions above the true. Such a relationship is exemplified in the 3-Mode eigenvalue graph and to a slightly lesser extent in the INDSCAL graph of variance explained (see Figure 1). Since a sizeable difference in the measures exists between dimensions 1 and 2, 2 and 3, and 3 and 4, but not 4 and 5, or any higher dimensional solutions, a 3-dimensional solution is strongly suggested. The INDSCAL measure of recovery does not pinpoint the 3-dimensional structure as vividly as does the 3-Mode measure.

Although the recoverability parameter plottings closely resemble the "classical" 3-dimensional solution ("elbow" in the curve), the results must not be accepted without question. A prime criterion is evaluating the dimensionality of the derived configuration rests upon the interpretability of the final solution. Therefore, the 4-dimensional and 2-dimensional solutions were examined as alternatives to the 3-dimensional configuration. Few of the theoretically relevant attributes such as expected return, dividend yield, variability of returns, etc., however, were closely related with the resultant axes of the 4-dimensional or 2-dimensional INDSCAL spaces. In addition, the individual-by-individual analyses of the INDSCAL correlation measures of recovery for the 2-, 3-, and 4-dimensional solutions supported the 3-dimensional representation. For example, except for few individuals, the recovery correlations did not increase appreciably between the 3- and 4-dimensional solutions. The correlations did increase noticeably, however, between the

Figure 1

Percent of Variance Explained by Dimension of Potential Investment Stock Space

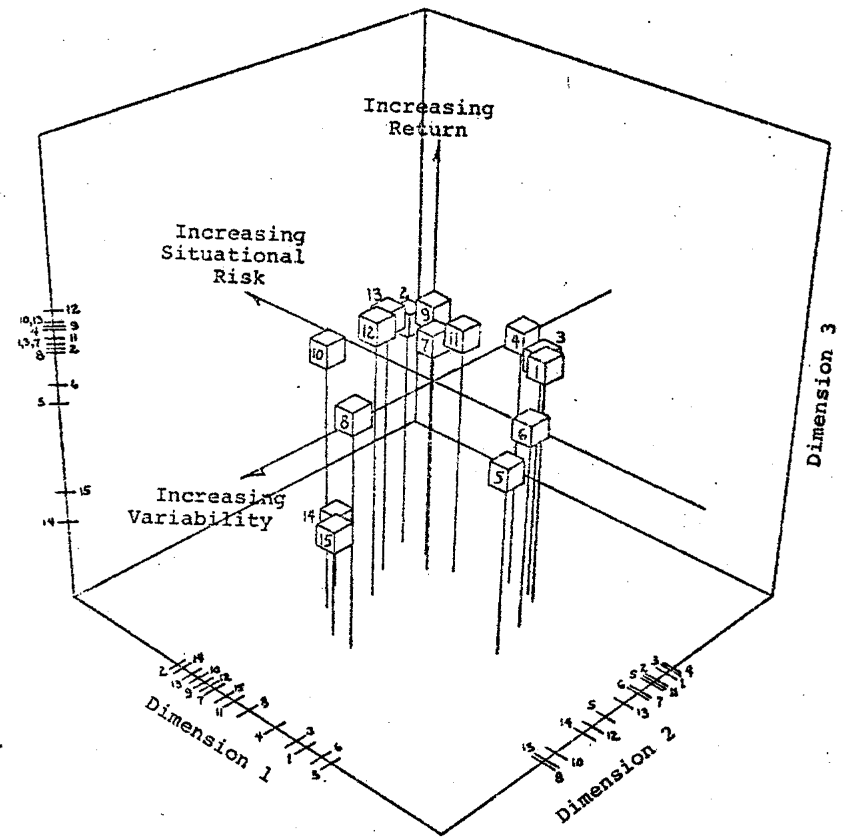




2- and 3-dimensional representation. The INDSCAL 3-dimensional stock space is shown in Figure 2.

Before initiating the statistical property fitting procedures to identify the 3 dimensions, average ratings for the attribute scales incorporated in the questionnaire were calculated. In addition, values for each of the 15 stocks were taken from Value Line (May, 1974) for the following attributes: 1) price stability; 2) beta; 3) price-earnings ratio; 4) dividend yield; and 5) growth persistence. The identification of the dimensions derived in INDSCAL mapping entailed statistically relating the stock ratings along the 24 attributes (19 respondent rated and 5 investment service) with the derived configuration. These 24 attribute vectors were fit into the INDSCAL stock space (see Figure 3) according to a multiple regression procedure, PROFIT (Carroll, 1968). The regression procedure formulates the stock rating attribute vectors (dependent variables) as a function of the stock coordinates along the dimensions (independent variables) of the derived space. Standardization of the regression coefficients reveals the direction cosines between the attribute vectors and the perceptual dimensions. The advantage of the multiple regression procedure over the frequently used simple correlation procedure is that not only are direction cosines derived which locate the unidimensional scales in the derived configuration, but the multiple correlation coefficients also measure how well the attributes fit into the space. The correlations from this linear regression of the attributes and the stock coordinates were all significant at the five percent level. Tables 3 and 4 report the property fitting results. The clustering of the attributes about each of the dimensions (see Figure 3 and Table 4) complemented by the

Figure 2: Stock Space



Stocks

1. IBM
2. ATT
3. Eastman Kodak
4. Sears
5. Polaroid
6. Avon
7. Aetna Life
8. National Airlines
9. Exxon
10. National Steel
11. Beatrice Foods
12. St. Regis Paper
13. General Motors
14. Con. Edison N.Y.
15. LTV Corp.

Table 3: Correlations between the Common Stock Attributes and the Stock Projections onto the Fitted Vectors in the Stock Space

Attribute Number	Correlation	Attribute
1	0.769	Government Influence
2	0.865	Interest Rate Sensitivity
3	0.921 *	Growth Persistence *
4	0.931	Dividend Yield
5	0.971	Trading Profit Potential
6	0.927	Consistency in Earnings Growth
7	0.862	Liquidity
8	0.933	Business Risk
9	0.954	Variation of Returns
10	0.796 *	Price Stability *
11	0.744 *	Beta *
12	0.973	Possibility of Loss
13	0.794	Predictability of Future Earnings
14	0.952	Price Volatility
15	0.972	Riskiness
16	0.980	Credit Risk
17	0.875 *	Price Earning Ratio *
18	0.877 *	Dividend Yield *
19	0.967	Percentage Return
20	0.955	Outlook for Firm
21	0.927	Outlook for Industry
22	0.968	Price Appreciation Potential
23	0.958	Management
24	0.864	Market Potential for Firm's Goods

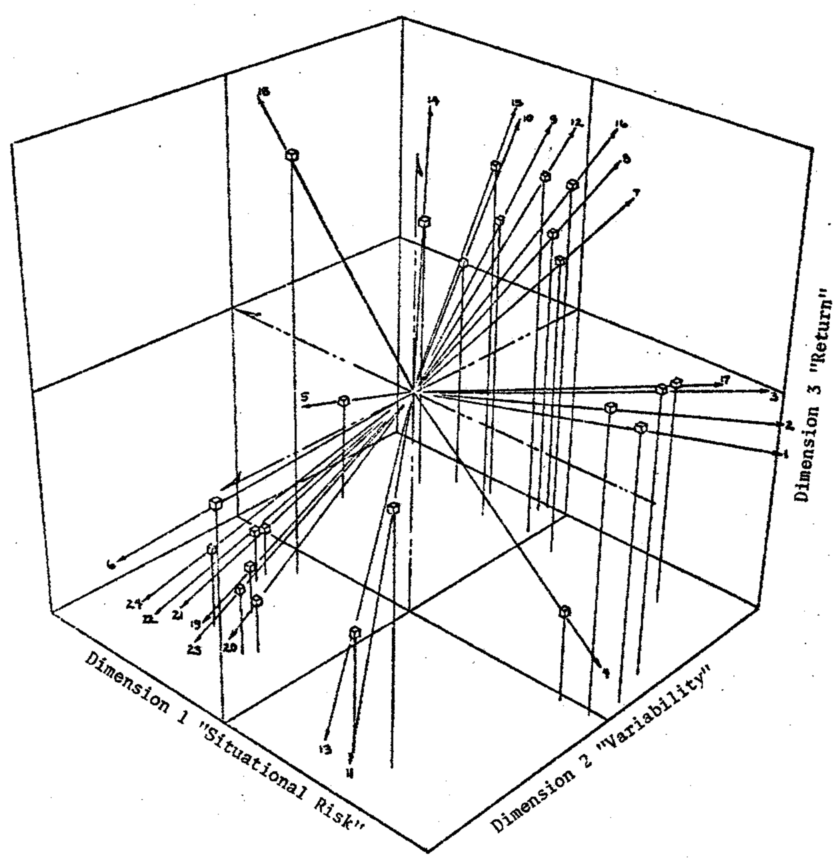
* Variables from Value Line

Table 4 : Direction Cosines of Fitted Vector in Normalized Stock Space

No.	Attribute Description	Dimension 1 Situational Risk	Dimension 2 Variability	Dimension 3 Return
1	Government Influence	-0.974	-0.103	0.199
2	Interest Rate Sensitivity	-0.943	0.046	0.328
3	Growth Persistence *	-0.922	-0.287	0.259
4	Dividend Yield	-0.805	0.040	-0.590
5	Trading Profit Potential	0.764	-0.406	0.500
6	Consistency in Earnings Growth	0.014	0.998	-0.052
7	Liquidity	0.089	-0.976	0.194
8	Business Risk	0.110	-0.941	0.319
9	Variation of Returns	0.338	-0.884	0.320
10	Price Stability *	0.510	-0.856	0.078
11	Beta *	-0.617	0.778	0.112
12	Possibility of Loss	0.026	-0.767	0.641
13	Predictability of Earnings	-0.415	0.751	-0.512
14	Price Volatility	0.621	-0.734	0.274
15	Riskiness	0.248	-0.733	0.633
16	Credit Risk	-0.146	-0.721	0.676
17	Price Earning Ratio *	-0.693	-0.720	0.024
18	Dividend Yield *	0.508	0.129	0.851
19	Percentage Return	0.441	0.448	-0.777
20	Outlook for Firm	0.229	0.616	-0.753
21	Outlook for Industry	0.594	0.225	-0.771
22	Price Appreciation Potential	0.584	0.291	-0.757
23	Management	0.249	0.679	-0.690
24	Mkt. Pot. for Firm's Goods	0.456	0.630	-0.627

* Variables from Value Line

Figure 3: Attributes in Stock Space



areas of interest reflected in the respondent reported criteria suggested the following labeling of the three dimensions:

1. situational risk (macro, external, and dynamic factors);
2. variability (the more micro firm oriented factors); and
3. expected return (income and price appreciation).

COMPARISON OF THE 3-MODE AND THE INDSCAL SOLUTIONS

Individual differences scaling analysis develops not only an object space but also a person space. Therefore, the comparison of the solutions from the two models will be twofold: first, the relationship between the two derived object spaces will be examined, and secondly, the relationship between the person spaces will be examined.

Since the 3-Mode object space is not uniquely determined, the comparison of the two spaces was accomplished by rotating the 3-Mode space toward the uniquely determined INDSCAL space. The 3-dimensional matrices were rotated via a Procrustes transformation (Kaiser, 1960). If we define the 3-Mode object space as matrix "A" and the INDSCAL object space as matrix "B," then in simplified terms the Procrustes transformation derives the matrix "T" which minimizes the sum of the squared discrepancies "E" between the elements of the transformed 3-Mode matrix and the INDSCAL matrix, i.e.,

$$AT = B + E.$$

The transformation matrix and the discrepancy matrix from the comparison of the 3-dimensional 3-Mode and INDSCAL object spaces are reported in Table 5 the largest element in the discrepancy matrix is only 0.026. These very small discrepancies clearly demonstrate that the INDSCAL solution is very

Table 5

Summary Matrices for Procrustes Comparison of INDSCAL and 3-Mode Derived Spaces

TRANSFORMATION MATRIX

	1	2	3
1	-0.578	-0.873	0.664
2	-0.797	0.179	-0.530
3	0.172	-0.452	-0.526

DISCREPANCY MATRIX

		1	2	3
IBM	1	-0.018	0.023	0.012
ATT	2	0.021	-0.019	0.000
Eastman Kodak	3	-0.012	0.006	0.000
Sears	4	-0.019	0.019	0.010
Polaroid	5	0.017	-0.012	-0.007
Avon	6	0.015	-0.012	0.002
Aetna Ins.	7	-0.005	-0.012	-0.012
National Airlines	8	0.005	-0.000	0.002
Exxon	9	0.026	-0.021	-0.026
National Steel	10	-0.014	0.014	0.020
Beatrice Foods	11	0.008	-0.007	-0.001
St. Regis Paper	12	-0.015	0.018	0.007
GM	13	0.002	-0.000	-0.006
Con. Ed. of N.Y.	14	-0.010	0.022	0.020
LTV Corporation	15	-0.000	-0.018	-0.022

UNROTATED CORE MATRIX

	1	2	3
1	4.156	0.016	0.007
2	0.016	2.732	0.009
3	0.007	0.009	1.420

TRANSFORMED CORE MATRIX

	1	2	3
1	3.001	-0.107	0.017
2	-0.107	2.950	0.057
[illegible]	[illegible]	[illegible]	[illegible]

nearly identical to the more general 3-Mode solution. Congruence of the two solutions following rotation confirms that the INDSCAL solution is not due to a local minimum.

Examination of the person space eigenvalues (Table 2) from the 3-Mode analysis reveals that one person dimension dominates the explanation of the individuals' perception of the stocks. The orientation of the object dimensions as perceived by the first person dimension is reflected in the core matrix. Since the object space is 3-dimensional, examination of the first 3 X 3 section of the core matrix will reveal the general orientation of the object dimensions. The unrotated and rotated core matrices are presented in Table 5 (rotation utilized the transformation matrix "T" which brought the object spaces from the two models into agreement, i.e., $G^T = (T^{-1} X T^{-1}) G$ where X denotes the Kronecker product). The first observation that can be made with regard to the rotated first person core matrix is that it is nearly diagonal. Since the off-diagonal entries reflect the degree of obliqueness in perceptual orientation of the object dimensions, the absence of sizeable off-diagonal elements reveals that the dominant first person orientation (general perception among the portfolio managers) is orthogonal. This orthogonality among object dimensions is a fundamental assumption of the INDSCAL model. Since the more general 3-Mode model has independently developed an object space which is nearly identical with the more easily interpretable INDSCAL space, we have confirmed the accuracy of the INDSCAL object space.

Next, the 3-Mode 3-dimensional person matrix and the INDSCAL 3-dimensional weight matrix were compared using the same procedure as that used in examining the object spaces from the two models. A Procrustes rotation was applied to the 1-dimensional 3-Mode person space (first person dimension) in rotating it toward the INDSCAL 3-dimensional weight matrix. The transformation matrix and the discrepancy matrix are reported in Table 6. Also included in Table 6 is the mean of the absolute value of the discrepancies for each dimension. Although the discrepancies are noticeably larger than the discrepancies found in comparing the object spaces, the fit still appears to be reasonable. Since there are no statistics to guide the interpretation of the closeness of fit, the discrepancies from the 1-dimensional rotation were compared with the discrepancies from the 2- and 3-dimensional rotations. Examination of the 2-dimensional and 3-dimensional rotation of the 3-Mode person space toward the 3-dimensional INDSCAL weight matrix discloses that the first dimension of the 3-Mode person space explains about as much of the variation in the INDSCAL weight matrix as does the three dimensions (see Tables 7 and 8). This observation suggests that the weights from the INDSCAL analysis should not be used as if they were independent variables. Interpretation of the weights as independent variables may result in misleading conclusions of perceptual differences.

If much of the information about the weight matrix is contained in one person space dimension of the 3-Mode analysis, then the weight matrix of the INDSCAL analysis must be less than full rank. The rank of the INDSCAL weight matrix was examined by calculating the eigenvalues of a cross product matrix formed from the INDSCAL weight matrix (Table 9). The large first eigenvalue and the relatively small second and third eigenvalues confirms that while the weight matrix may be 3-dimensional the first dimension clearly dominates as indicated by 3-Mode. Thus, the variation in perception may be more concisely represented with a single index for each individual.

Table 6

Procrustes Transformation Matrix for Rotating the 1-Dimensional 3-Mode Person Space Toward the INDSCAL 3-Dimensional Weight Matrix

	1	2	3
1	2.957	2.824	2.447

Discrepancies Between the 3-Dimensional INDSCAL Weight Matrix and the Procrustes Rotation of the 1-Dimensional 3-Mode Person Space

	1	2	3
1	-0.043	-0.042	0.028
2	-0.004	-0.102	0.102
3	-0.265	0.164	0.125
4	-0.198	-0.085	0.043
5	0.159	0.058	0.103
6	-0.066	0.081	-0.002
7	-0.204	0.342	0.035
8	0.124	-0.178	-0.070
9	-0.157	0.068	-0.048
10	-0.027	0.048	-0.055
11	0.039	-0.188	0.000
12	0.016	-0.122	-0.035
13	-0.211	0.089	0.030
14	0.233	-0.216	0.019
15	-0.002	0.156	-0.094
16	-0.056	0.253	-0.203
17	0.109	-0.220	0.149
18	0.287	0.174	-0.260
19	0.059	0.089	-0.139
20	0.109	-0.014	-0.106
21	0.224	-0.044	-0.076
22	0.055	-0.153	0.094
23	-0.037	-0.091	0.028
24	0.123	-0.240	0.170
25	-0.017	0.155	-0.029
26	0.267	-0.101	-0.309
27	-0.159	-0.067	0.134
28	-0.169	0.162	0.003
29	-0.137	-0.091	0.174
30	0.021	-0.156	0.223
31	-0.158	0.176	0.048
32	-0.075	-0.217	0.003
33	0.038	-0.153	0.088
34	-0.039	-0.020	0.184
35	-0.130	0.063	-0.027
36	-0.183	-0.101	0.354
37	0.103	0.287	-0.034
38	0.192	-0.163	-0.061
39	0.187	-0.294	0.158
40	-0.085	0.240	-0.085
41	-0.160	0.038	0.219
42	-0.103	0.128	-0.143
43	0.200	-0.095	0.025
44	0.062	0.261	-0.489
45	0.079	0.067	-0.046
46	-0.100	0.109	-0.078
47	0.088	0.001	-0.130
48	-0.214	-0.010	0.185
49	-0.120	-0.181	0.194
50	0.132	0.065	-0.249
51	0.020	0.103	-0.209
52	0.156	-0.097	0.018
53	-0.000	0.022	-0.022

Mean Value of the Absolute Discrepancies

0.117	0.129	0.112

Table 7

Procrustes Transformation Matrix for Rotating the 2-Dimensional 3-Mode Person Space Toward the INDSCAL 3-Dimensional Weight Matrix

	1	2	3
1	2.957	2.824	2.446
2	0.242	0.183	-0.550

Discrepancies Between the 3-Dimensional INDSCAL Weight Matrix and the Procrustes Rotation of the 2-Dimensional 3-Mode Person Space

	1	2	3
1	-0.025	-0.028	-0.013
2	0.021	-0.083	0.044
3	-0.257	0.170	0.106
4	-0.201	-0.088	0.052
5	0.174	0.069	0.070
6	-0.079	0.070	0.028
7	-0.194	0.349	0.012
8	0.118	-0.182	-0.057
9	-0.184	0.048	0.011
10	-0.047	0.033	-0.010
11	0.017	-0.205	0.050
12	0.011	-0.126	-0.023
13	-0.290	0.029	0.209
14	0.210	-0.234	0.072
15	-0.043	0.125	-0.000
16	-0.100	0.220	-0.103
17	0.114	-0.216	0.138
18	0.279	0.167	-0.240
19	0.074	0.100	-0.173
20	0.104	-0.018	-0.094
21	0.152	-0.098	0.087
22	0.098	-0.121	-0.002
23	-0.038	-0.091	0.030
24	0.154	-0.216	0.098
25	0.033	0.193	-0.144
26	0.291	-0.083	-0.363
27	-0.137	-0.051	0.086
28	-0.166	0.164	-0.002
29	-0.098	-0.062	0.086
30	0.046	-0.137	0.165
31	-0.084	0.232	-0.120
32	-0.093	-0.231	0.046
33	-0.006	-0.187	0.189
34	-0.022	-0.007	0.144
35	-0.122	0.069	-0.044
36	-0.177	-0.097	0.342
37	0.093	0.279	-0.010
38	0.234	-0.131	-0.158
39	0.185	-0.295	0.162
40	-0.074	0.248	-0.110
41	-0.104	0.080	0.093
42	-0.107	0.125	-0.132
43	0.179	-0.111	0.074
44	-0.006	0.209	-0.332
45	0.042	0.039	0.037
46	-0.082	0.122	-0.117
47	0.064	-0.016	-0.075
48	-0.154	0.034	0.049
49	-0.074	-0.146	0.088
50	0.080	0.025	-0.131
51	-0.013	0.077	-0.130
52	0.151	-0.101	0.029
53	-0.001	0.021	-0.019

Mean Value of the Absolute Discrepancies

1	2	3
0.111	0.126	0.098

Table 8

Procrustes Transformation Matrix for Rotating the 3-Dimensional 3-Mode Person Space Toward the INDSCAL 3-Dimensional Weight Matrix

	1	2	3
1	2.957	2.824	2.446
2	0.242	0.183	-0.550
3	-0.017	0.144	-0.275

Discrepancies Between the 3-Dimensional INDSCAL Weight Matrix and the Procrustes Rotation of the 3-Dimensional 3-Mode Person Space

	1	2	3
1	-0.025	-0.028	-0.014
2	0.020	-0.073	0.027
3	-0.255	0.161	0.122
4	-0.202	-0.086	0.048
5	0.176	0.055	0.095
6	-0.082	0.092	-0.012
7	-0.192	0.332	0.044
8	0.119	-0.189	-0.043
9	-0.188	0.086	-0.061
10	-0.044	0.008	0.037
11	0.021	-0.239	0.116
12	0.007	-0.093	-0.087
13	-0.289	0.021	0.225
14	0.212	-0.250	0.103
15	-0.045	0.136	-0.020
16	-0.102	0.235	-0.133
17	0.112	-0.193	0.094
18	0.278	0.174	-0.254
19	0.070	0.135	-0.240
20	0.107	-0.041	-0.051
21	0.150	-0.085	0.061
22	0.098	-0.121	-0.002
23	-0.035	-0.116	0.076
24	0.154	-0.215	0.097
25	0.030	0.213	-0.181
26	0.294	-0.109	-0.313
27	-0.139	-0.035	0.057
28	-0.168	0.178	-0.028
29	-0.099	-0.058	0.079
30	0.046	-0.142	0.175
31	-0.084	0.237	-0.130
32	-0.090	-0.255	0.091
33	-0.009	-0.162	0.142
34	-0.018	-0.033	0.194
35	-0.118	0.033	0.023
36	-0.181	-0.071	0.293
37	0.092	0.288	-0.028
38	0.237	-0.155	-0.111
39	0.180	-0.252	0.079
40	-0.070	0.223	-0.062
41	-0.103	0.067	0.118
42	-0.105	0.106	-0.097
43	0.176	-0.093	0.039
44	-0.003	0.180	-0.277
45	0.043	0.030	0.055
46	-0.081	0.107	-0.088
47	0.063	-0.013	-0.081
48	-0.154	0.033	0.050
49	-0.077	-0.124	0.046
50	0.080	0.024	-0.129
51	-0.012	0.064	-0.105
52	0.150	-0.087	0.003
53	-0.001	0.020	-0.016

Mean Value of the Absolute Discrepancies

	1	2	3
	0.111	0.124	0.097

Table 9

Decomposition of the Cross-Product Matrix Formed from the 3-Dimensional INDSCAL Weight Matrix

Eigenvalues	Percent Variance	Cumulative Percent
22.78	87.76	87.76
1.75	6.75	94.51
1.42	5.49	100.00

Factor Matrix

	1	2	3
1	0.791	-0.045	0.023
2	0.702	-0.140	-0.005
3	0.675	0.014	0.333
4	0.714	-0.085	0.130
5	0.513	-0.036	-0.057
6	0.670	0.058	0.087
7	0.601	0.198	0.334
8	0.669	-0.058	-0.212
9	0.730	0.084	0.134
10	0.592	0.077	0.020
11	0.678	-0.118	-0.122
12	0.752	-0.048	-0.087
13	0.653	0.037	0.218
14	0.645	-0.150	-0.280
15	0.561	0.176	0.036
16	0.578	0.322	0.080
17	0.677	-0.252	-0.132
18	0.545	0.316	-0.249
19	0.619	0.167	-0.061
20	0.575	0.075	-0.137
21	0.564	0.033	-0.228
22	0.709	-0.167	-0.080
23	0.716	-0.076	-0.004
24	0.632	-0.281	-0.143
25	0.567	0.127	0.074
26	0.679	0.174	-0.383
27	0.751	-0.142	0.145
28	0.702	0.106	0.209
29	0.666	-0.188	0.132
30	0.629	-0.267	-0.002
31	0.631	0.081	0.225
32	0.732	-0.140	-0.044
33	0.685	-0.162	-0.069
34	0.591	-0.150	0.093
35	0.672	0.065	0.119
36	0.695	-0.332	0.235
37	0.482	0.216	0.039
38	0.671	-0.054	-0.254
39	0.583	-0.307	-0.224
40	0.539	0.223	0.145
41	0.546	-0.139	0.230
42	0.741	0.196	0.082
43	0.533	-0.077	-0.193
44	0.682	0.546	-0.122
45	0.567	0.083	-0.050
46	0.727	0.134	0.097
47	0.715	0.105	-0.122
48	0.722	-0.144	0.235
49	0.741	-0.261	0.085
50	0.681	0.236	-0.175
51	0.717	0.231	-0.053
52	0.623	-0.072	-0.163
53	0.688	0.035	0.000

CONCLUSION

The INDSCAL model offers a readily accessible and easily interpretable and understood solution of individual differences scaling data. However, one must be made aware at the outset of the restrictions implicit in the underlying assumptions of the model's formulation. Comparison of the solution derived from a 3-Mode scaling with the INDSCAL derived solution allows a direct examination of several of these assumptions and provides insight into the accuracy of the simpler INDSCAL solution. For the stock perception study, analysis by both models confirmed the accuracy of the orthogonal assumption, the absence of the local minimum problem, and provided insight into the correct dimensionality of the solution. The 3-Mode analysis also revealed dependence among the dimensional weights. Direct analysis of the INDSCAL object space provided meaningful interpretation of the factors utilized by institutional investors in assessing investment in common stocks. Direct interpretation of the object space is a very positive feature of the INDSCAL model, but the relevance and applicability of the model must be examined in order to minimize the presence of technique artifacts in the analyzed data.

REFERENCES

Carroll, J. Douglas and Jih-Jie Chang. "Analysis of Individual Differences in Multidimensional Scaling Via An N-Way Generalization of 'Eckart-Young' Decomposition," Psychometrika, Vol. 35, (September, 1970).

Chang, J.J., and J.D. Carroll. "How to Use PROFIT, A Computer Program for Property Fitting by Optimizing Nonlinear or Linear Correlation," Bell Laboratories, Unpublished Manuscript, 1968.

Green, Paul E., and Frank J. Carmone. Multidimensional Scaling and Related Techniques in Marketing Analysis, Boston: Allyn and Bacon, 1970.

Kaiser, H.F. "Relating Factors Between Studies Based Upon Different Individuals," unpublished MS., University of Illinois, 1960.

MacCallum, Robert C. "Relations Between Factor Analysis and Multidimensional Scaling," Psychological Bulletin, Vol. 81, No. 8 (August, 1974).

Ray, Robert M. BLOCKS: A Fortran IV Program for Plotting Planar Projections of Three-Dimensional Block Models, University of Illinois, June, 1972.

Ross, R.T. "Optimum Orders for the Presentation of Pairs in the Method of Paired Comparisons," Journal of Educational Psychology, Vol. 25 (1934).

Sheth, Jagdish N. "The Multivariate Revolution in Marketing Research," Journal of Marketing, Vol. 35 (January, 1971).

Sheth, Jagdish N. "How to Get the Most Out of Multivariate Methods," Faculty Working Paper, College of Commerce and Business Administration, University of Illinois, April, 1975.

Torgerson, W.S. Theory and Methods of Scaling, New York: John Wiley and Sons, Inc., 1958.

Tucker, Ledyard R. "Relations Between Multidimensional Scaling and Three-Mode Factor Analysis," Psychometrika, Vol. 37 (March, 1972).