

The Foundations of Factor Analysis,

Stanley A. Mulaik. New York: McGraw-Hill Book Company, 1972. xvi+453 pp. \$14.95.

This is a delightful addition to the limited number of books devoted exclusively to the topic of factor analysis in multivariate statistics. It is rare to come across a book in multivariate analysis which engrosses the reader the same way as does a good book in history. The main reason for the pleasant reading experience, of course, is due to the historical, almost biographical account of factor analysis which the author portrays in the book.

According to the author, the emphasis of the book is algebraic rather than statistical. The objective of the book is to provide the reader with the mathematical rationale for factor-analytic procedures. It is designed as a text for students of the behavioral or social sciences at the graduate level.

The book consists of sixteen chapters. Predictably, the first chapter is an introduction to factor analysis. In less than fifteen pages, the author beautifully provides a philosophical, historical and conceptual framework for factor analysis. The second chapter, which is rather long, deals with basic concepts of matrix algebra and calculus. It is simply useless because it is too elementary for a graduate course in factor analysis. It is a puzzle why every textbook in multivariate analysis ends with one or more chapters devoted to matrix algebra which the reader already knows. Perhaps it is to provide notational security or introduce to the reader the notational idiosyncrasies of the author.

The third chapter is somewhat unique and extremely useful as a prelude to the fundamental axioms in multivariate analysis, namely the creation of composite variables or linear compounds from manifest variables. The chapter provides a very good though elementary treatment of equally and unequally weighted linear composites in both the equation and the matrix forms.

Chapter 4 is a brief refresher in multiple and partial correlation. While it is well written, I failed to see its need in the book especially at this point in the book.

Chapters 5 and 6 provide the mathematical basis of factor analysis. These two chapters are, however, among the weakest in the book in terms of exposition and smooth transition of thoughts. The author describes numerous methods of factor extraction including the diagonal method, the Cholesky method, the centroid method, principal axes methods, Hotelling's iterative method, the Jacobi method, tridiagonalization methods and the Q-R method. It is simply too many methods for the average reader who is anyhow likely to depend on some canned statistical program which probably utilizes some combination of principal axes and Jacobi methods. One almost feels a sigh of relief when the last page of the chapter is in sight. In other words, information overload is real! To add to the problem, the author seems to lose the reader's perspective when he either provides too much or too little information about various methods (Jacobi method too much and centroid method too little). Finally, there is no evaluative judgment by the author about numerous methods except his concern for rounding errors in the computation. The only section which saves this chapter from becoming dull is the excellent treatment of principal axes methods.

From this low point, the book rises again as a fascinating reading experience in Chapters 7 and 8 which concern themselves with the common factor and other types of factor analysis. The treatment of the communality problem, Guttman's solutions for the lower bound estimates of the communality, the Minres procedure, the Fletcher-Powell algorithm and its modifications by Joreskog, and finally maximum likelihood procedures is extremely lively and readable with just the sufficient amount of historical anecdotes and perspectives. Similar treatments of principal components analysis (unweighted and weighted), image analysis, alpha factor analysis, and canonical factor analysis fill up the contents of Chapter 8. The only fault one can find is the obvious and almost emotional love for Guttman expressed by the author. It seems as

if he is trying hard to ensure that Guttman's contribution to factor analysis is fully recognized in the history.

Chapters 9-12 deal with the rotation of factor axes. Chapter 9 describing graphical rotation is unfortunately a useless filler despite the author's desire to provide a feel for concreteness which the reader may not otherwise get in the computerized world. The chapter fails to provide that feeling of concreteness. There seems no good substitute as yet for the classical writings of Thurstone (1935) and Burt (1941) in this regard. The variety of orthogonal analytical rotations including the Quartimax, the Varimax, the Transvarimax, and the Parsimax are well documented in Chapter 10. Similarly, the oblique analytic rotations such as the Oblimax, the Oblimin, and the Maxplane criteria are fully discussed in Chapter 11. It was, however, somewhat surprising to find that one full chapter is allocated to Procrustes rotations, especially when the author also has a separate chapter on confirmatory factor analysis later in the book which has close conceptual if not statistical parallel with Procrustes procedures. The discussion of Procrustes rotations, however, is extremely readable and good.

Another low point in the book is reached when factor scores are discussed in Chapter 13. It has always been a puzzle to this reviewer as to why American psychologists have always given factor scores less than adequate treatment. The popular book by Harman in its first edition (1960) had very little to say, and even the revised edition (1967) has only a chapter in contrast to almost five to seven chapters on factor pattern and factor structure. Mulaik's book also suffers from a similar imbalance of treatment between the factor loadings and the factor scores aspects. This is even more puzzling in the current setting in which factor analysis (component or common factor) is extensively used as an intermediate step in cluster analysis for reducing the rank of the correlated profile variables to their orthogonal structural dimensions. Finally, the author himself justifies factor analysis as a basis for developing structural theories which examine the aggregate of highly correlated elemental components or system of variables.

Chapters 14-16 discuss tangential issues in factor analysis, such as the problem of factor, invariance, hypothesis testing (confirmatory) factor analysis, and the relationship of factor analysis to regression, canonical correlations and discriminant analysis. The best chapter is one on confirmatory factor analysis, which is unique to the book.

No textbook is perfect and this one is no exception. There seem to be three aspects where the book could have been improved. First, there is almost a total lack of empirical examples of large scale studies from the behavioral or social sciences. There is not a single real world example in the four critical chapters which enumerate different methods of factor analysis. This reviewer could not help chuckle when the author states (p. 226):

Let us move from the abstract to an empirical example. Suppose one administers several tests of addition and multiplication of numbers to young elementary school children who have learned addition and multiplication by a process of memorizing what number is to be associated with a given pair of numbers under a given operation.

If this is an empirical example, I am at a loss what to call data from a large scale study conducted in a natural setting. It would have been extremely advantageous to have a common data bank on which various methods of factor analysis are applied so that the student could see and feel the impact on the structure which different methods make in factor analysis. In this regard, the book is decidedly weaker than Rummel (1970) and the revised edition of Harman (1967).

A second weakness in the book relates to the uneven treatment given to several topics, including Procrustes rotations and Guttman's lowest bound estimate of communality. A related matter is the omission of several interesting topics such as nonlinear, nonmetric and multimode factor analysis which the author himself states are left out in order to meet the limitations of time and space. It would appear that the book would have become more comprehensive if chapters on matrix algebra, multiple correlation and graphic rotation were replaced with these topics of factor analysis.

The third weakness is the algebraic treatment of factor analysis to the almost complete exclusion of sampling theory. No matter how we look at it, no textbook will be considered complete without providing statistical tests of significance of the parameters based on the sampling theory of multivariate normal distributions.

This reviewer also believes that the author consciously attempted to maximize the contributions by Guttman and Johnson and mini-

mize the contributions by C.R. Rao and Eckart-Young. But then this is a free country!

The book does an admirable job in minimizing typographical and notational errors. Out of more than five hundred equations, I could find only two errors: first, on p. 179 in equation (8-14), the subscript should be j and not i in one place; and second, on p. 334, in equation (13-21), the term (U^{-1}) is missing without explanation or reason. It is a delight to see that the publisher and the author have taken extra care to minimize such errors.

In summary, I think the book clearly deserves a coveted place on the shelf of books on factor analysis. It fills an important gap by providing an up-to-date historical-mathematical perspective. This aspect is not adequately treated in either Harman (1967) or in Rummel (1970). Perhaps the best thing about the book is the engrossing manner in which the author narrates what is otherwise a dull and difficult subject.

JAGDISH N. SHETH
*University of Pittsburgh and University of
Illinois at Urbana-Champaign*

REFERENCES

- [1] Burt, C.L., *The Factors of the Mind*, New York: The MacMillan Co., 1947.
- [2] Harman, H.H., *Modern Factor Analysis*, rev. ed., Chicago: University of Chicago Press, 1967.
- [3] Rummel, R.J., *Applied Factor Analysis*, Evanston, Ill.: Northwestern University Press, 1970.
- [4] Thurstone, L.L., *The Vectors of Mind*, Chicago: University of Chicago Press, 1935.